

Number Systems

Tuesday, January 4, 2022 11:10 AM

Key Bases: Decimal (10), Binary (2), Hex (16), Octal (8)

Binary \rightarrow Hex: Groups 4 bits

Binary \rightarrow Octal: Groups 3 bits

CMOS, nMOS, pMOS

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Basic Terminology

Voltage: Electric potential difference

Current: Flow of charged particles $\frac{\partial Q}{\partial t}$, always from (+) \rightarrow (-)

Ohm's Law: $V = IR$, in general $V = IZ$

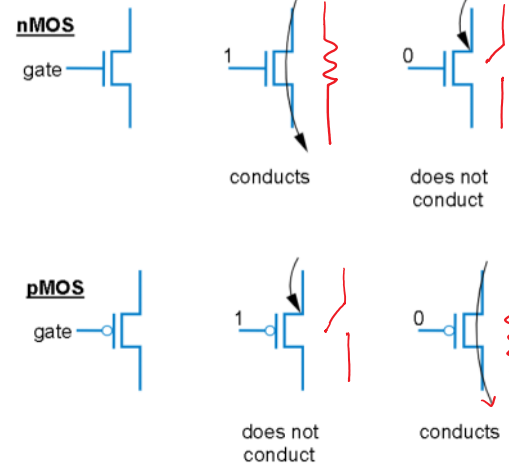
CMOS Switch

Complimentary Metal Oxide Semiconductor:

↳ Negative: conducts when gate is 1

↳ Positive: conducts when gate is 0

When closed, switch acts as a resistor

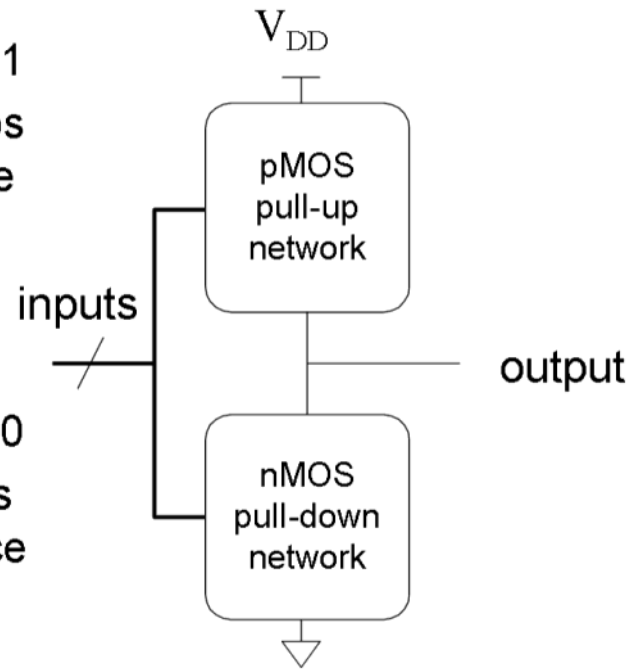


- **nMOS:**

- Turns on when gate is connected to 1
- When turned on, nMOS passes zeros well, but not ones, so connect source to GND
- nMOS forms a pull-down network


- **pMOS:**

- Turns on when gate is connected to 0
- When turned on, pMOS passes ones well, but not zeros, so connect source to V_{DD}
- pMOS forms a pull-up network



Common Logic Gates


AND - 6 CMOS



a	b	AND
0	0	0
0	1	0
1	0	0
1	1	1

$a \cdot b$


OR - 6 CMOS



a	b	OR
0	0	0
0	1	1
1	0	1
1	1	1

$a+b$

NOT - 2 CMOS



a	NOT
0	1
1	0

a'

BUFFER - 4 CMOS




a	BUF
0	0
1	1

a

NOT
NOR
NAND } basic CMOS gates

- Derived operators:


NAND - 4 CMOS



a	b	NAND
0	0	1
0	1	1
1	0	1
1	1	0

$(a \cdot b)'$

NOR - 4 CMOS



a	b	NOR
0	0	1
0	1	0
1	0	0
1	1	0


$(a+b)'$

XOR - 8 CMOS



a	b	XOR
0	0	0
0	1	1
1	0	1
1	1	0

XNOR - 8 CMOS



a	b	XNOR
0	0	1
0	1	0
1	0	0
1	1	1

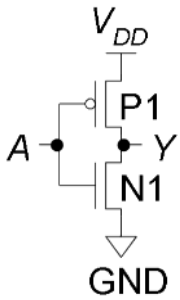
Axioms and Theorems

Axiom	Dual	Name
A1 $B = 0$ if $B \neq 1$	A1' $B = 1$ if $B \neq 0$	Binary field
A2 $\bar{0} = 1$	A2' $\bar{1} = 0$	NOT
A3 $0 \cdot 0 = 0$	A3' $1 + 1 = 1$	AND/OR
A4 $1 \cdot 1 = 1$	A4' $0 + 0 = 0$	AND/OR
A5 $0 \cdot 1 = 1 \cdot 0 = 0$	A5' $1 + 0 = 0 + 1 = 1$	AND/OR

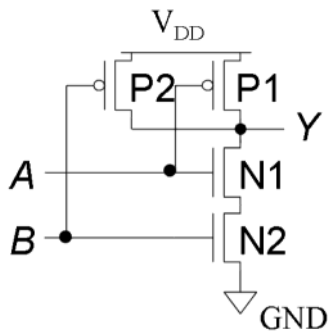
Theorem	Dual	Name
T1 $B \cdot 1 = B$	T1' $B + 0 = B$	Identity
T2 $B \cdot 0 = 0$	T2' $B + 1 = 1$	Null Element
T3 $B \cdot B = B$	T3' $B + B = B$	Idempotency
T4 $\bar{\bar{B}} = B$		Involution
T5 $B \cdot \bar{B} = 0$	T5' $B + \bar{B} = 1$	Complements

Theorem	Dual	Name
T6 $B \cdot C = C \cdot B$	T6' $B + C = C + B$	Commutativity
T7 $(B \cdot C) \cdot D = B \cdot (C \cdot D)$	T7' $(B + C) + D = B + (C + D)$	Associativity
T8 $(B \cdot C) + B \cdot D = B \cdot (C + D)$	T8' $(B + C) \cdot (B + D) = B + (C \cdot D)$	Distributivity
T9 $B \cdot (\bar{B} + C) = B$	T9' $B + (B \cdot C) = B$	Covering
T10 $(B \cdot C) + (B \cdot \bar{C}) = B$	T10' $(B + C) \cdot (B + \bar{C}) = B$	Combining
T11 $(B \cdot C) + (\bar{B} \cdot D) + (C \cdot D) = B \cdot C + \bar{B} \cdot D$	T11' $(B + C) \cdot (\bar{B} + D) \cdot (C + D) = (B + C) \cdot (\bar{B} + D)$	Consensus
T12 $\overline{B_0 \cdot B_1 \cdot B_2 \dots} = (\bar{B}_0 + \bar{B}_1 + \bar{B}_2 \dots)$	T12' $\overline{B_0 + B_1 + B_2 \dots} = (\bar{B}_0 \cdot \bar{B}_1 \cdot \bar{B}_2 \dots)$	De Morgan's Theorem

NOT Gate:



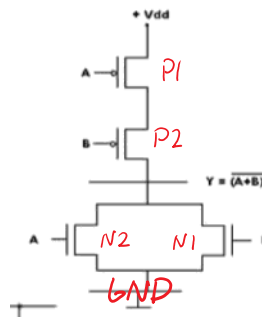
NAND Gate:



PMOS in Parallel

NMOS in Series

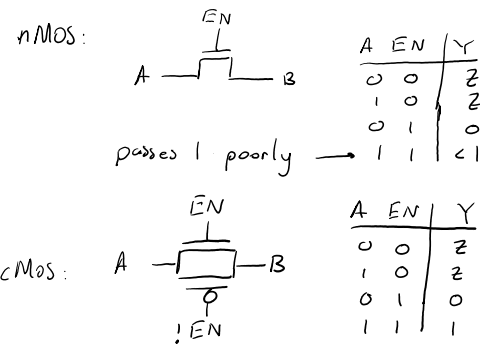
NOR Gate:



PMOS in Series

NMOS in Parallel

Transmission Gate:



nMOS:

A	EN	Y
0	0	Z
1	0	Z
0	1	0
1	1	1

passes 1 poorly →

cMOS:

A	EN	Y
0	0	Z
1	0	Z
0	1	0
1	1	1

Reason: Control when a signal is connected.

Delay: Delay is linear to resistance x capacitance. Going through a transistor has resistance and input into a transistor gate has capacitance.

For multiple stages, delay is additive

Bubble Pushing

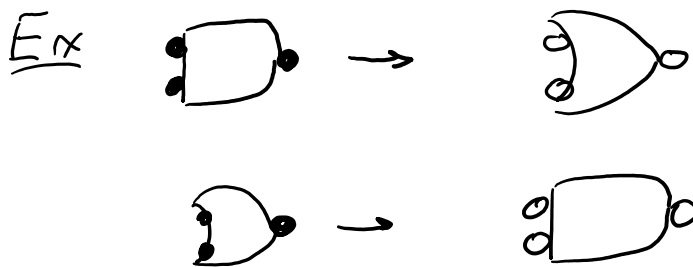
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1) Convert AND \leftrightarrow OR

2) Convert $\bullet \leftrightarrow \circ$

3) Combine \circ until simplified.

Uses DeMorgan's Laws to simplify circuits.



Notations, Canonical Form

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Complement: variable with bar or '

Literal: variable or its complement

Implicant: product of literals (AND)

Implicate: sum of literals (OR)

Minterm: product that includes all input variables

Maxterm: sum that includes all input variables

Canonical form: a sum of either minterms or maxterms, not a minimal form

Sum of Products Canonical form: sum of minterms: $\sum m$

$$F = 001 \quad 011 \quad 101 \quad 110 \quad 111$$

$$F = A'B'C + A'BC + AB'C + ABC' + ABC$$

A	B	C	F	F'
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$$F' = A'B'C' + A'BC' + AB'C'$$

Product of Sums Canonical form: product of maxterms: $\prod M$

$$F = 000 \quad 010 \quad 100$$

$$F = (A + B + C) (A + B' + C) (A' + B + C)$$

A	B	C	F	F'
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Converting: if $F = \sum m$ then $F = \prod M$ where M are the maxterms that are not used to define F in $\sum m$

Terms for 2-Level Simplification, Kmap Simplification

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- **Implicant**: Any element of ON-set or DC-set, **Implicate**: Any element of OFF-set or DC-set
- **Prime**: largest group of implicants/implicates that are powers of 2
- **Essential Prime**: a Prime that alone covers an element of ON-set

Def Algorithm:

- 1) Find Prime Implicants
- 2) Filter only the Essential Prime Implicants to minimize size and terms
- 3) Create minterms or maxterms according to the Prime Implicants

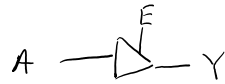
Note Don't cares can form primes but not essential primes.

Specifically: If a don't care can be used to form a larger group with another implicate, then it is included in a prime.

A don't care by itself will not form a prime.

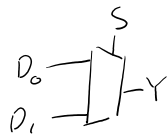
Basic Elements

• Tristate Buffer:



E	A	Y
0	0	Z
0	1	Z
1	0	0
1	1	1

• 2:1 Multiplexer



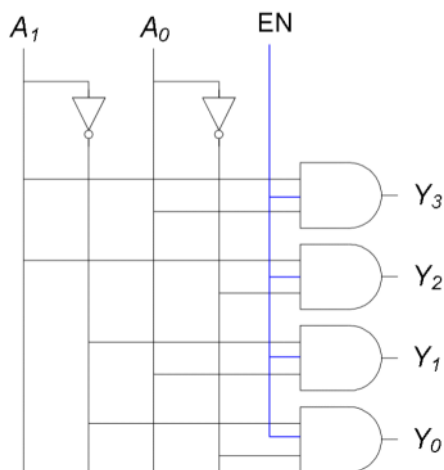
selects between 2 inputs

• N:1 Mux : selects between N inputs, $N = 2^k$

Idea We can use muxes as general purpose logic

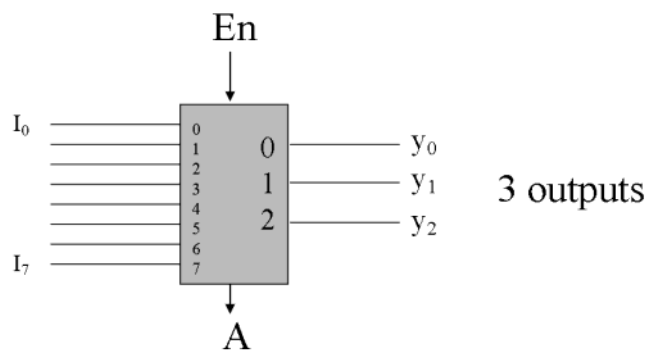
• Demultiplexers: given an input x and selector inputs. Connects x to specific output based on selector. If EN is 0, then outputs are 0

• Decoder: given inputs x , turns corresponding output 1 if $x = \text{output number}$ used for addressing memory/peripherals, all other outputs are 0



can be used to implement general logic element with the same number of inputs
Each output implements a minterm, use additional gates to implement logic

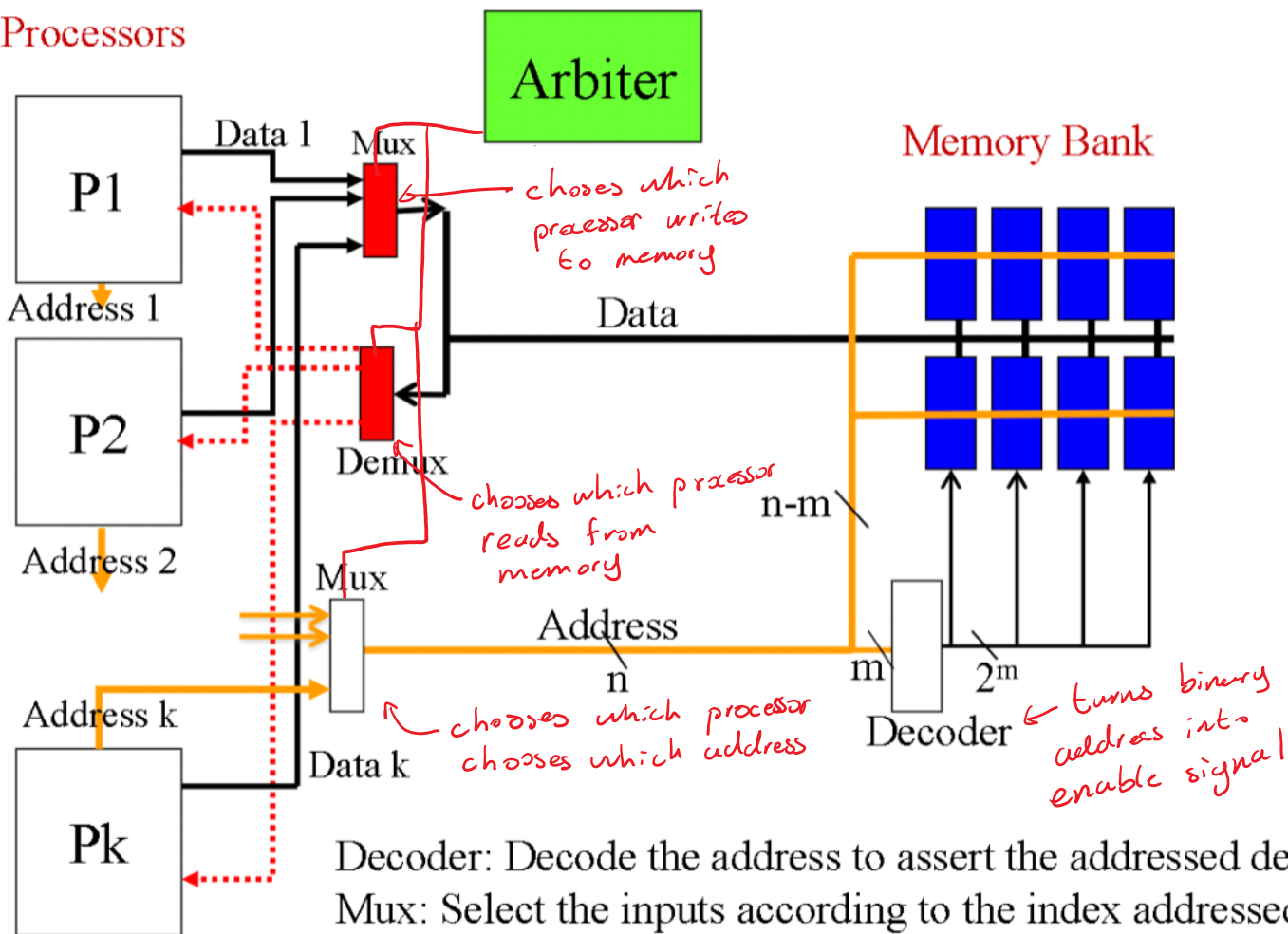
• Encoder: takes inputs and sets corresponding output to 1, else 0:



Basic Processor Design

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Processors

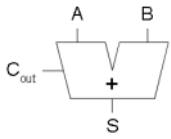


Decoder: Decode the address to assert the addressed device
Mux: Select the inputs according to the index addressed by the control signals

Adders (Ripple Carry, Carry Lookahead)

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Half Adder

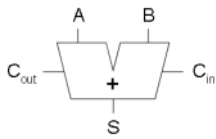


A	B	C _{out}	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$$S = A \oplus B$$

$$C_{out} = AB$$

Full Adder



C _{in}	A	B	C _{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$S = A \oplus B \oplus C_{in}$$

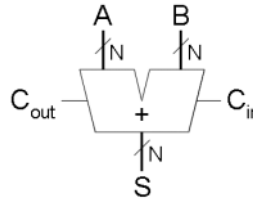
$$C_{out} = AB + AC_{in} + BC_{in}$$

Types of multi-bit adders

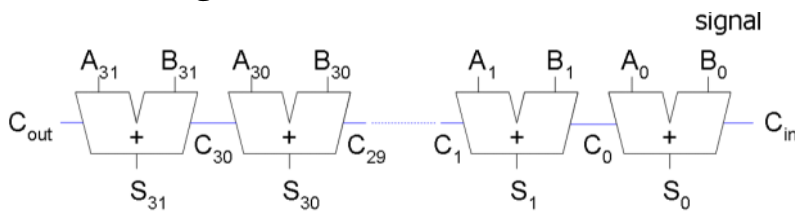
- Ripple-carry (slow)
- Carry-lookahead (faster)

Def N - number of bits to be added

Symbol



• Ripple Carry Adder: chains multiple FA together in a chain:

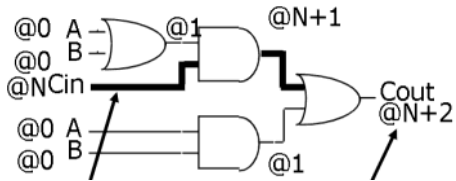


signal

Disadvantage! SLOW!

$$\text{Delay: } t_{ripple} = N \cdot t_{FA}$$

where N is the number of bits and t_{FA} is the gate delay of C_{out}

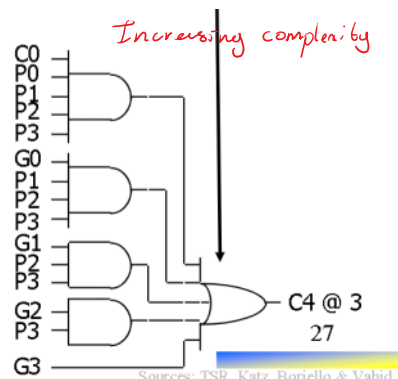
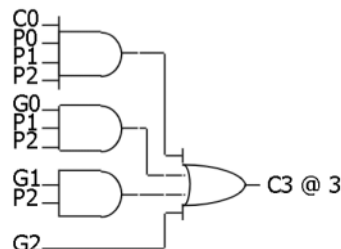
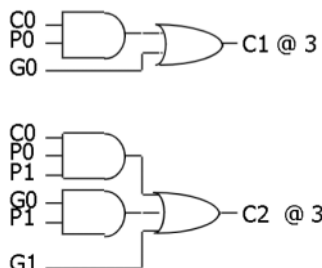
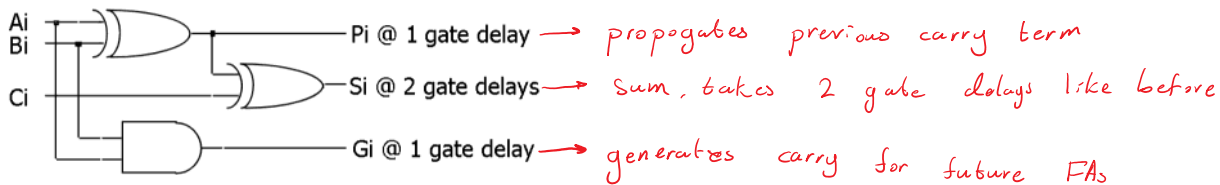


In this FA, the (out gate delay is 2 gate delays,

$$\text{so } t_{FA} = 2 \text{ (gate delays)}$$

$$\text{and } t_{ripple} = N \cdot 2 \text{ (gate delays)}$$

• Carry Lookahead Adder: Uses FA with propagate and generate inputs



Sources: TSR, Katz, Boriello & Vahid

Generally:

Disadvantage: increasing complexity as N increases

- **Step 1:** Compute G_i and P_i for all columns
- **Step 2:** Compute G and P for k -bit blocks
- **Step 3:** C_m propagates through each k -bit propagate/generate block

$$G_{ij} = G_i + P_i (G_{i-1} + P_{i-1} (G_{i-2} + P_{i-2} G_j))$$

$$P_{ij} = P_i P_{i-1} P_{i-2} P_j \quad \sim$$

$$C_i = G_{ij} + P_{ij} C_{i-1}$$

Idea: We can combine ripple carry and carry lookahead adders to create a fast and small adder.

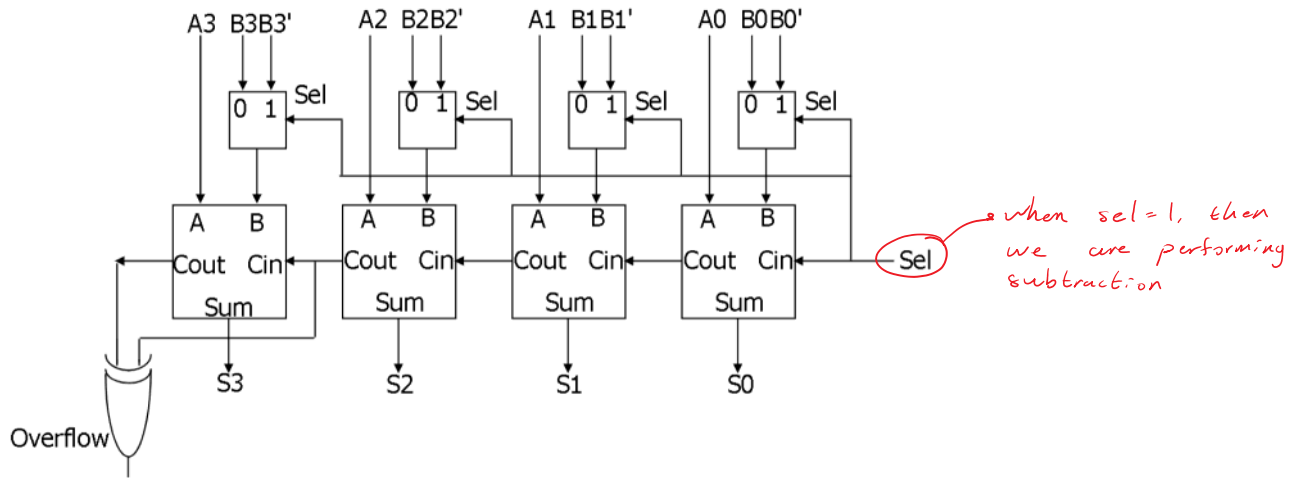
Subtractor (2's Complement)

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Idea: We can implement subtraction using 2's complement.

if we can perform $A + B$,

then we can perform $A - B$ with $A + \bar{B} + 1$



Detecting overflow:

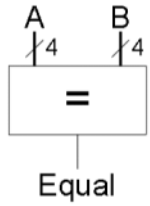
- 1) If the two input sign bits are the same but result sign bit is different
- 2) Difference between cin of sign bit and cout of sign bit
↳ simpler logic using 1 xor gate.

Comparators (Equal, Less Than, Greater Than)

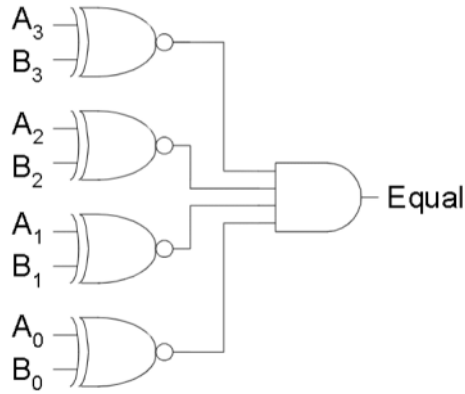
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Equality (comparator):

Symbol

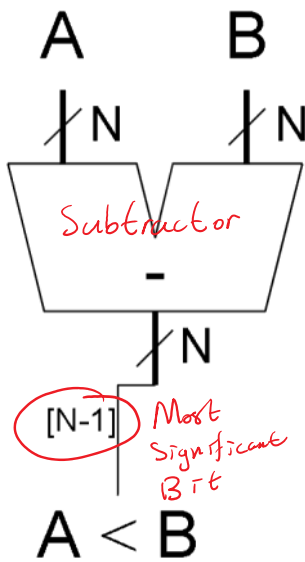


Implementation



Equal is only true when $A = B$

Less Than / Greater Than:



When output is 1, then $A < B$
When output is 0, then $A \geq B$

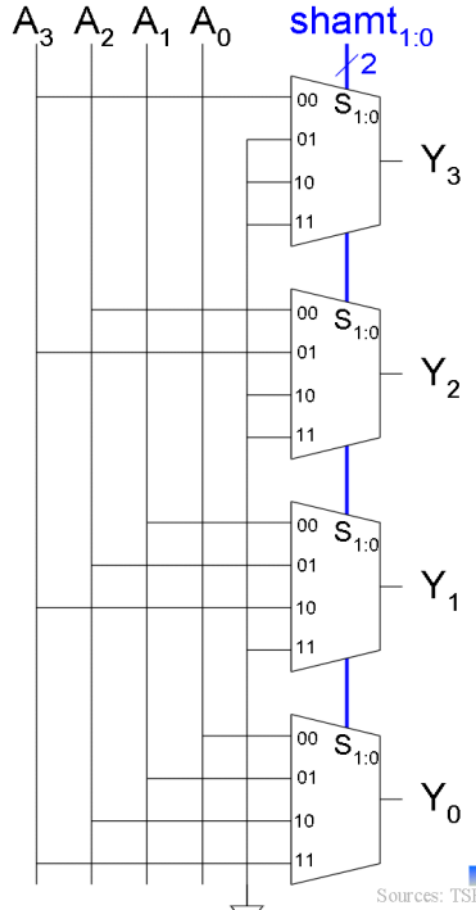
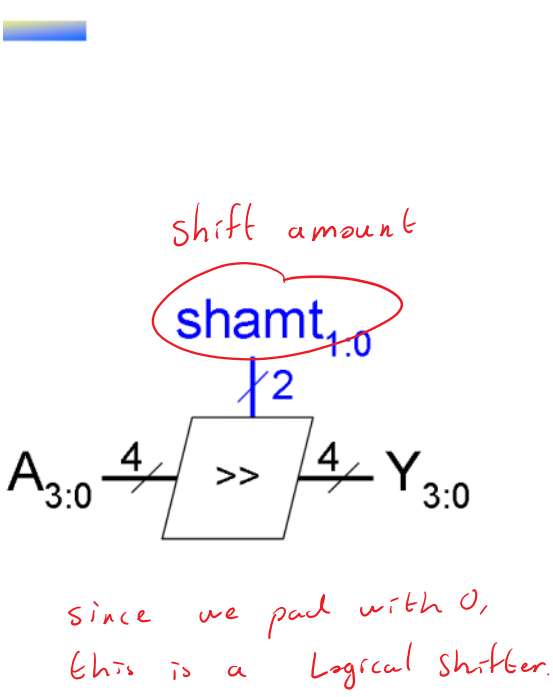
Shifters (Logical, Arithmetic, Rotator)

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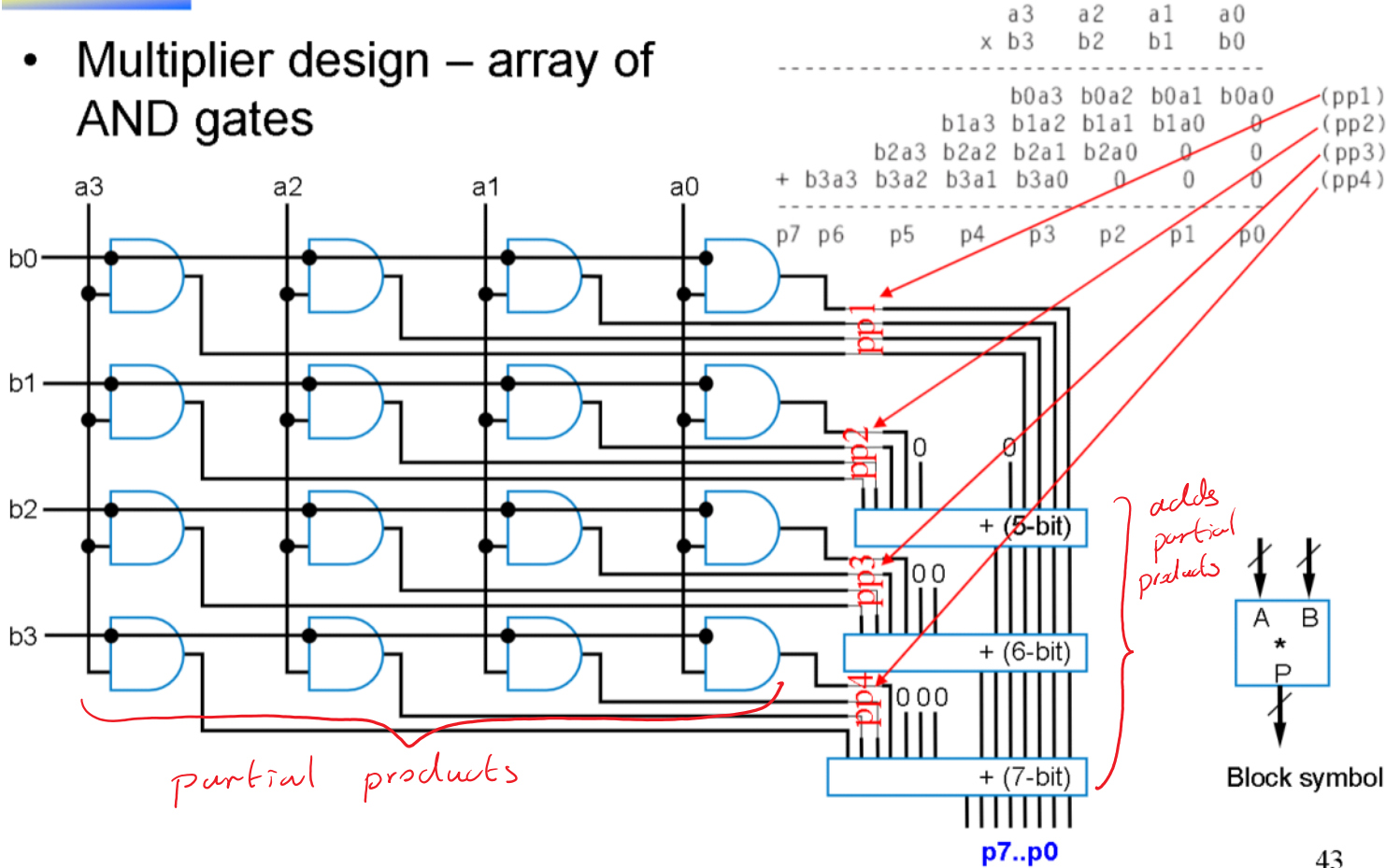
Logical Shifter: Fill empty spaces with 0

Arithmetic Shifter: On right shift, fill empty spaces with old MSB

Rotator: Wrap bits shifted off the end to new empty space



- Multiplier design – array of AND gates



Dividers

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Repeated subtraction

- Set quotient to 0
- Repeat while dividend \geq divisor
 - Subtract divisor from dividend
 - Add 1 to quotient
- When dividend $<$ divisor:
 - Remainder = dividend
 - Quotient is correct

Example:

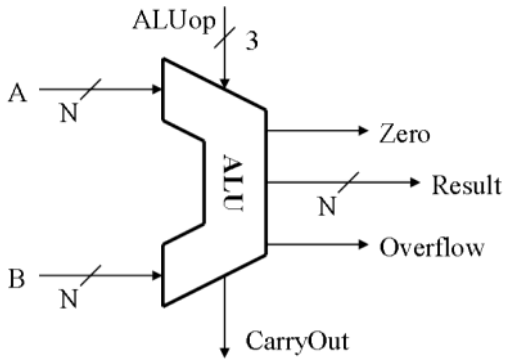
- Dividend: 101; Divisor: 10

Dividend		Quotient
101	-	0 +
10		1
11	-	1 +
10		1
1		10

ALU Design

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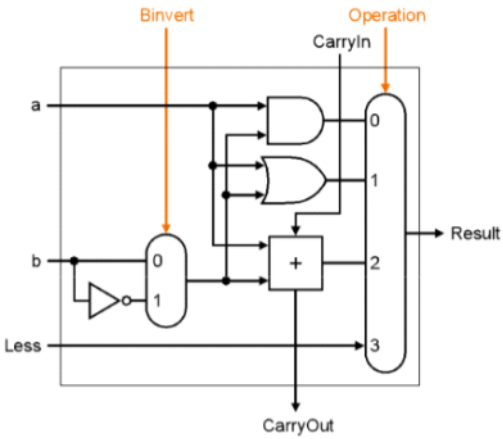
Block symbol:



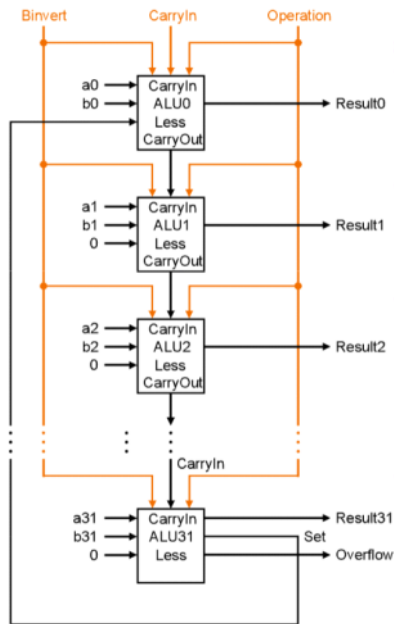
ALUop:

ALU Control Lines (ALUop)	Function
- 000	And
- 001	Or
- 010	Add
- 110	Subtract
- 111	Set-on-less-than

Implementation (1-bit):



Idea: We can chain together multiple 1-bit ALUs together to create larger ALUs capable of larger number operations.



Sequential Logic Components

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Idea: We want to create circuits to store data so that we can run a sequence of tasks.

Note: In computers: Registers \rightarrow Cache \rightarrow Memory \rightarrow Disks

Def: often we want to use a clock cycle to control sequential logic:

A synchronous circuit is a sequential circuit with a clock

Period: time between clock pulse starts

Cycle: time interval in periods

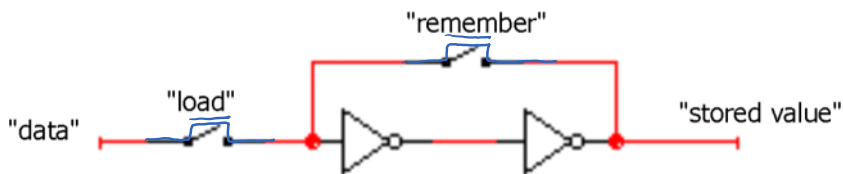
Duty Cycle: time spent in % that the clock is high

Frequency: $1/\text{period}$

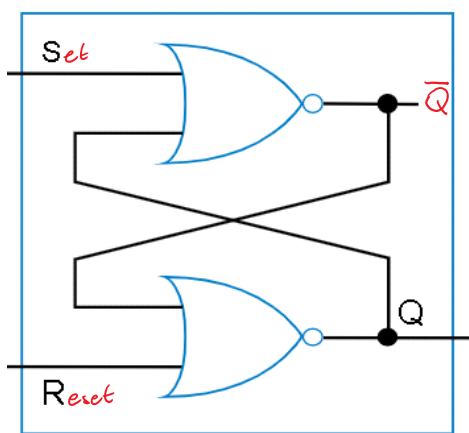
SRAM, SR Latch, D Latch

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SRAM: Simplest memory element, stores a value and recalls stored value



SR Latch: Set a value and hold until reset is activated

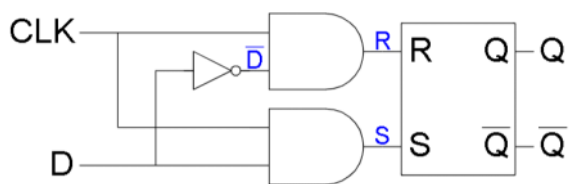


S	R	Q(t)	Q(t+Δ)	
0	0	0	0	hold
0	0	1	1	
0	1	0	0	reset
0	1	1	0	
1	0	0	1	set
1	0	1	1	
1	1	0	X	not allowed
1	1	1	X	

		S	
Q(t)	0	0	X
	1	0	X
		R	

characteristic equation
 $Q(t+\Delta) = S + R'Q(t)$

D Latch: Solves the illegal state in SR Latches, but must be refreshed



CLK	D	\bar{D}	S	R	Q	\bar{Q}	
0	X	\bar{X}	0	0	Q_{prev}	\bar{Q}_{prev}	← hold/memory
1	0	1	0	1	0	1	← reset
1	1	0	1	0	1	0	← set

The inverter and AND gates ensures R and S are never both 1

When clock goes high, it stores the value of D as Q

When clock goes low, it retains the previous value of Q

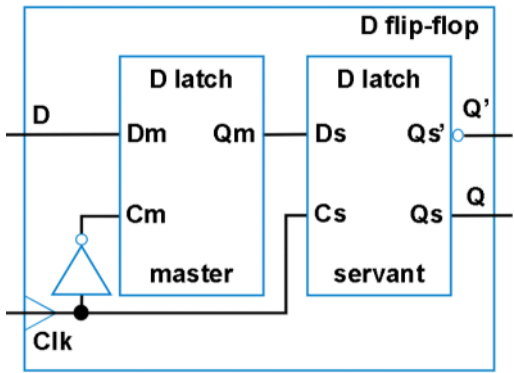
Summary! When clock = 1, D passes through Q (transparent)

When clock = 0, Q holds its previous value (opaque) (at the falling edge)

D Flip-Flop

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D Flip-Flop:



Master latch loads when $Clk = 0$

Servant latch loads when $Clk = 1$

Id	D	Q(t)	Q(t+1)
0	0	0	0
1	0	1	0
2	1	0	1
3	1	1	1

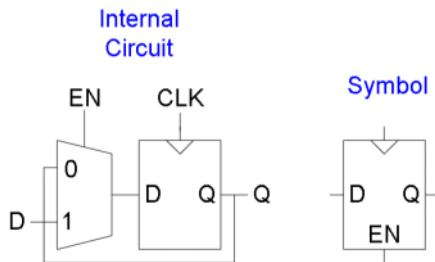
Thus for D Flip-Flop:

$$Q(t+1) = D(t)$$

The value of Q changes to D at every positive clock edge. Otherwise it holds the value.

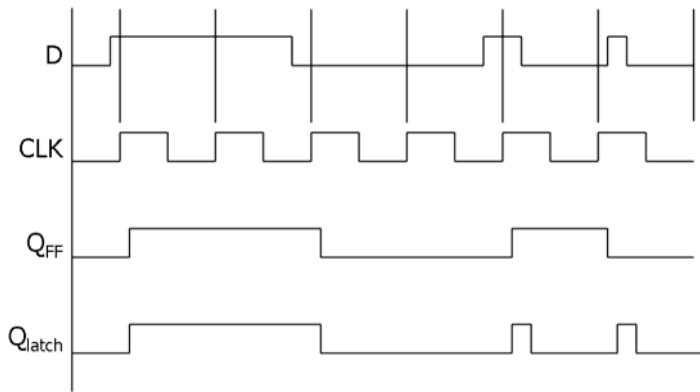
Adding an Enable:

- **EN = 1:** D passes through to Q on the clock edge
- **EN = 0:** the flip-flop retains its previous state



D Latch vs D Flip-Flop

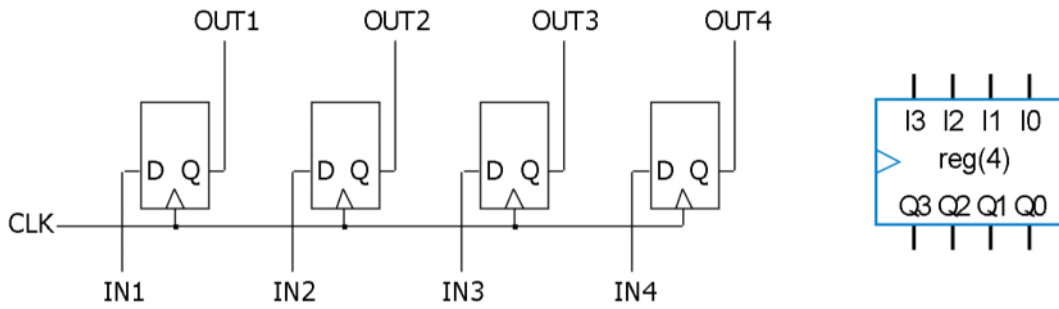
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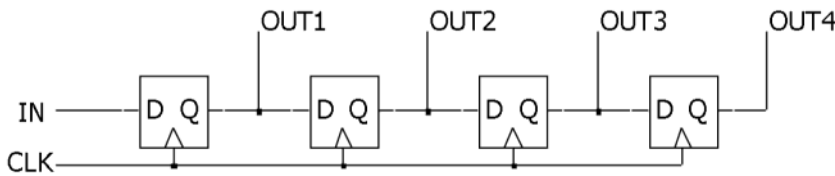
Flip Flop: takes D at \uparrow and holds value until next \uparrow

Latch: Copies D when CLK=1, holds previous value when CLK=0.

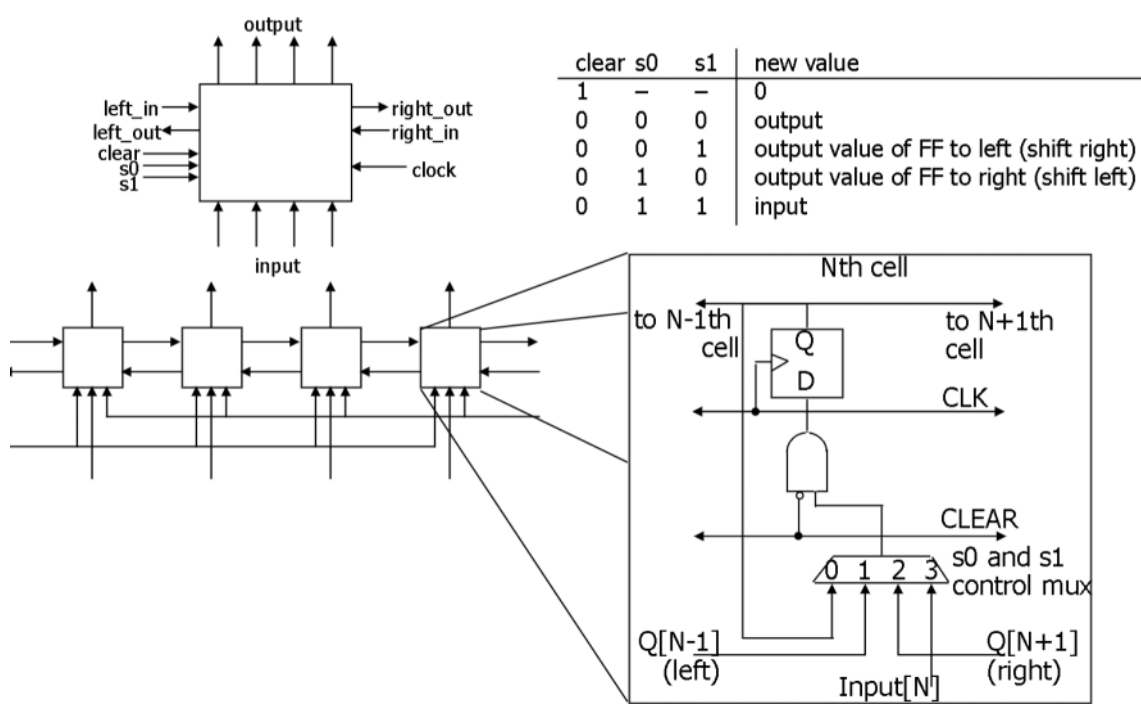
Basic Register (4-bit): Holds values for one clock cycle



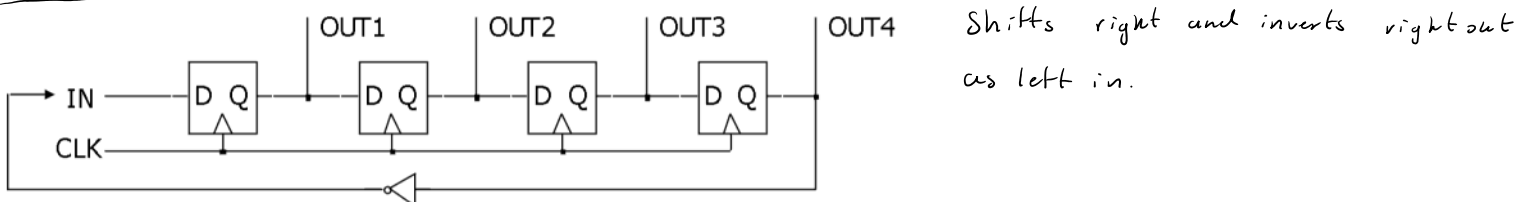
Shift Register: Holds and shifts values of consecutive samples



Universal Shift Register: Allows shifting in both directions



Counter: Sequences through a fixed set of patterns



Finite State Machine

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Idea: In combinational logic, we had no feedback.

In sequential logic, we have feedback and memory, which we can make finite state machines.

Def An FSM consists of:

- set of states
- set of inputs and outputs
- initial state
- set of transitions, only one can be true at a time

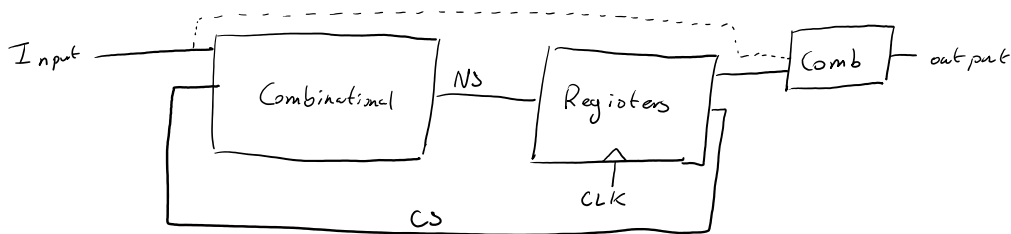
Notation: State Diagram: Graph where nodes are states and directed edges are transitions

State Table: Table showing each current state and next states

State Assignments: Binary representation of each state

Excitation Table: State Table but with state Assignments

Creating A Circuit: Excitation Table shows the combinational logic:



where NS is the next state
CS is the current state

Registers are D Flip-Flop(s)

Finding Combinational: Use k-maps, where each output/next state are outputs and inputs/current state are inputs.

Mealy Machine vs Moore Machine

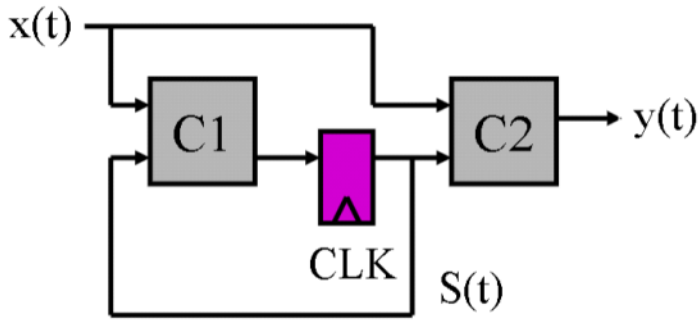
Friday, February 4, 2022 8:34 PM

Mealy Machine: $y_i(t) = f_i(X(t), S(t))$

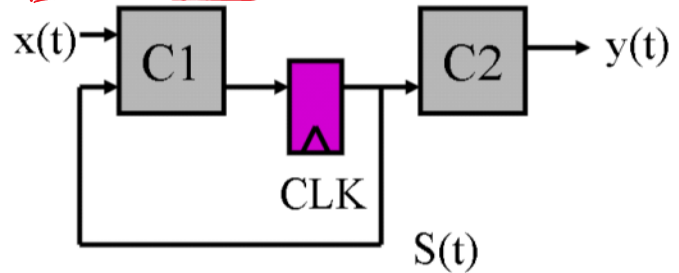
Moore Machine: $y_i(t) = f_i(S(t))$

Always:

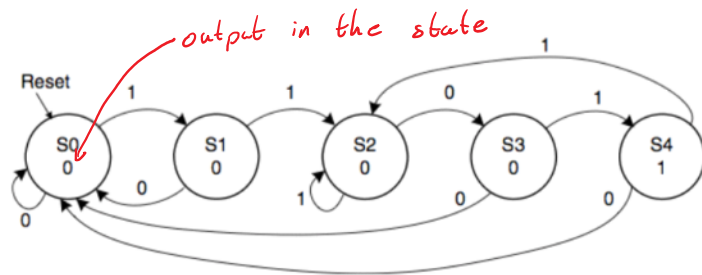
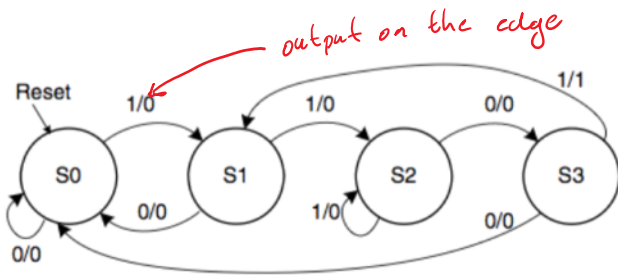
$$s_i(t+1) = g_i(X(t), S(t))$$



Mealy Machine



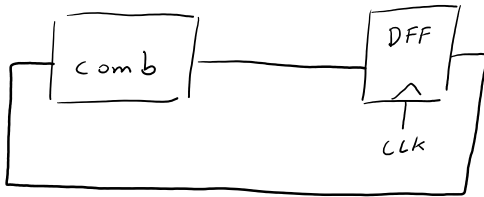
Moore Machine



In Mealy Machine: Output is function of inputs and state

In Moore Machine: Output is function of state only

Def Given the template FSM:



there are timing constraints on the FSM

Def: For combinational logic:

Min delay of a gate, also called contamination delay: t_{cd}

Minimum time from when an input changes until the output *starts* to change

Max delay of a gate, also called propagation delay: t_{pd}

Maximum time from when an input changes until the output *is* guaranteed to reach its final value (i.e., stop changing)

Def: For sequential logic:

On inputs: **Setup time: t_{setup}**

Time *before* the clock edge that data must be stable (i.e. not change)

Hold time: t_{hold}

Time *after* the clock edge that data must be stable

Aperture time: t_a

Time around clock edge that data must be stable ($t_a = t_{setup} + t_{hold}$)

On outputs:

Min delay of FF, also called contamination delay or min CLK to Q delay: t_{ccq}

Time after clock edge that Q might be unstable (i.e., starts changing)

Max delay of FF, also called propagation delay or maximum CLK to Q delay: t_{pcq}

Time after clock edge that the output Q is guaranteed to be stable (i.e. stops changing)

Def: For clock signal:

The clock doesn't arrive at all registers at the same time

Skew: difference between the two clock edges

Perform the **worst case analysis**

Idea: when picking a clock speed

$$T_{\text{clock}} \geq t_{\text{pcq}} + t_{\text{pd}} + t_{\text{setup}} + t_{\text{skew}}$$

$$t_{\text{hold}} < t_{\text{cd}} + t_{\text{ccq}} - t_{\text{skew}}$$

High Level State Machine

Sunday, February 13, 2022 2:35 PM

Idea Some behaviors may be too complex to describe using FSMs

Solution: Use a high level description for a theoretical FSM called High Level State Machine

Def A High Level State Machine extends FSM with:

- multiple bit input/outputs
- to cal storage
- arithmetic operations

Conventions:

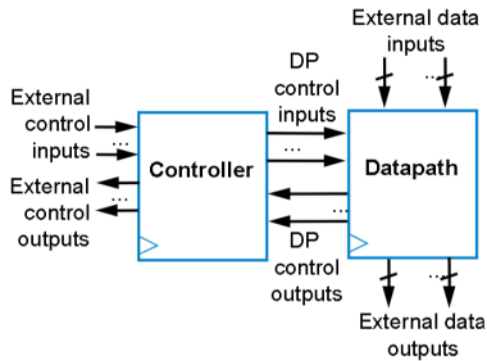
- **Numbers:**
 - Single-bit: '0' (single quotes)
 - Integer: 0 (no quotes)
 - Multi-bit: "0000" (double quotes)
- == for comparison equal
- Multi-bit outputs *must* be registered via local storage
- // precedes a comment

RTL design process:

Capture the behavior

Convert it to a circuit

- High-level architecture (datapath and control path)
- Datapath capable of HLSM's data operations
- Design controller to control the datapath



- Steps:
- 1) Draw the HLSM
 - 2) Determine what data will be stored and how it will be modified.
 - ↳ Design the Datapath (Sequential circuit)
 - 3) Determine how to control the Datapath, what states have what control signals
 - ↳ Design the Controller (FSM)

- Def:
- A Data dominant design: extensive datapath, simple controller
 - A Control dominant design: simple datapath, extensive controller

Idea: Convert each C construct to equivalent states and transitions.

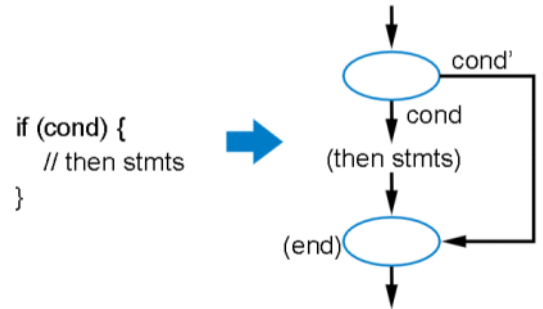
Assignment statement

- Becomes one state with assignment



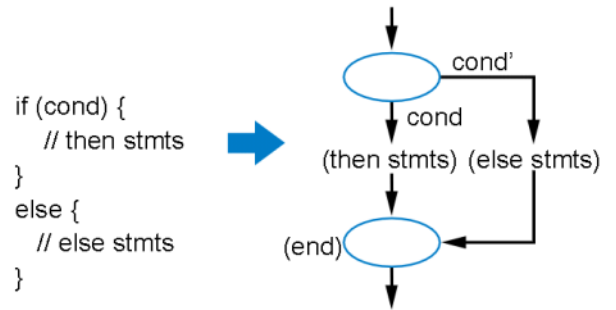
If-then statement

- Becomes state with condition check, transitioning to "then" statements if condition true, otherwise to ending state
- "then" statements would also be converted to states



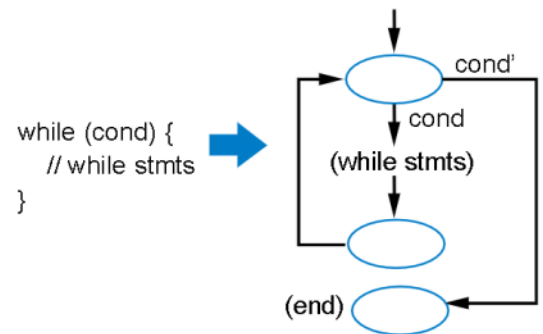
If-then-else

- Becomes state with condition check, transitioning to "then" statements if condition true, or to "else" statements if condition false



While loop statement

- Becomes state with condition check, transitioning to while loop's statements if true, then transitioning back to condition check



Memory, RAM vs ROM

Saturday, February 26, 2022 4:23 PM

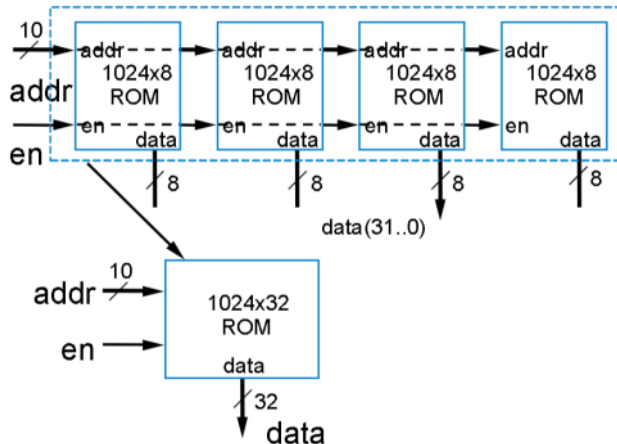
Idea: Stores a large number of bits

has n/w to select read or write

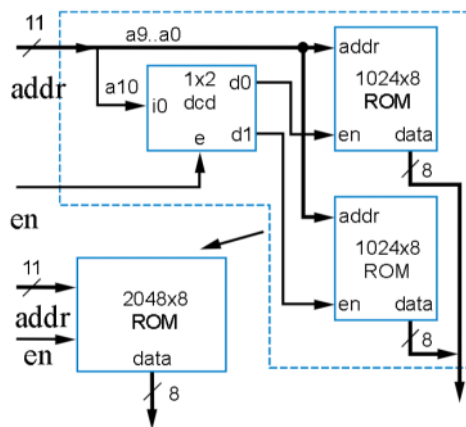
has enable to read/write when addressed

can have multipoint: multiple accesses to different addresses simultaneously

Combining: Wider words: connect memory in parallel:



Combining: More memory: connect memory in parallel with decoder:



Def Traditionally: ROM is read-only, bits stored without power

RAM is dynamic, bits lost without power

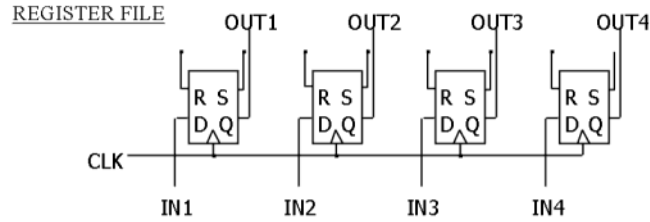
Lotely, distinctions blurred: Advanced ROMs can be written to (EEPROM)
Advanced RAMs can hold bits without power (NVRAM)

Types of RAM:

Memory is Volatile

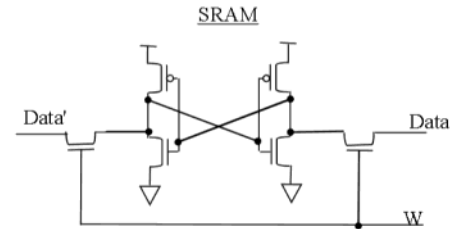
Register file

- Fastest
- But biggest size
- Used in registers*



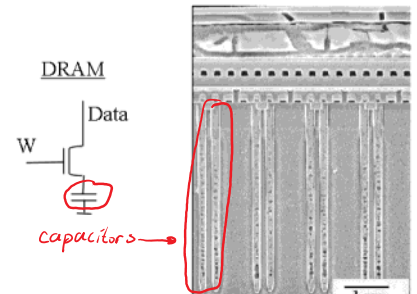
SRAM

- Fast (e.g. 10ns)
- More compact than register file
- Used in cache*



DRAM

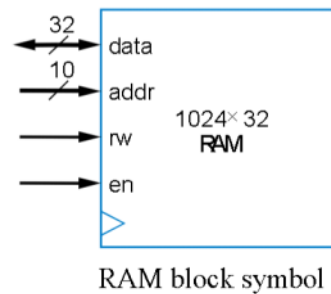
- Slowest (e.g. 20ns)
 - And refreshing takes time
- But very compact
- Different technology for large caps.
- Used in off-chip RAM*



Def Block symbol and characteristics of RAM:

RAM – Readable and writable memory

- Logically the same as register file
 - RAM just one port; register file two or more
- RAM vs. register file
 - RAM is larger
 - RAM stores bits using a bit storage vs. FFs
 - RAM implemented on a chip in a square – keeps longest wires (hence delay) short

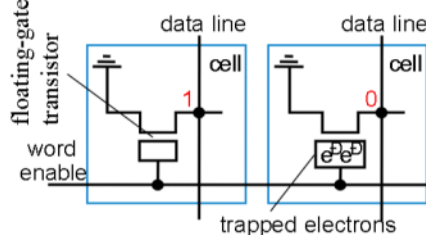


Idea: We want to store data without having to refresh the bits
 We can use ROMs

Def: Traditional ROMs use quantum tunneling to trap data in a floating gate. Use a strong voltage to free electron.

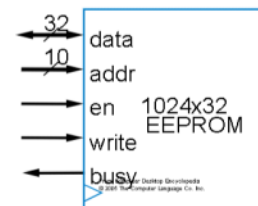
Erasable Programmable ROM (EPROM)

- Uses "floating-gate transistor" in each cell
- Programmer uses higher-than-normal voltage so electrons *tunnel* into the gate
 - Electrons become trapped in the gate
 - Only done for cells that should store 0
 - Other cells will be 1
- To erase, shine ultraviolet light onto chip
 - Gives trapped electrons energy to escape
 - Requires chip package to have window



Electrically-Erasable Programmable ROM (EEPROM)

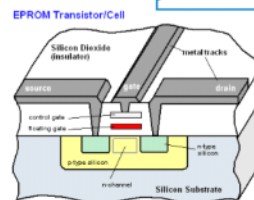
- Programming similar to EPROM
- Erasing one word at a time *electronically*



Flash memory

- Like EEPROM, but large blocks of words can be erased *simultaneously*

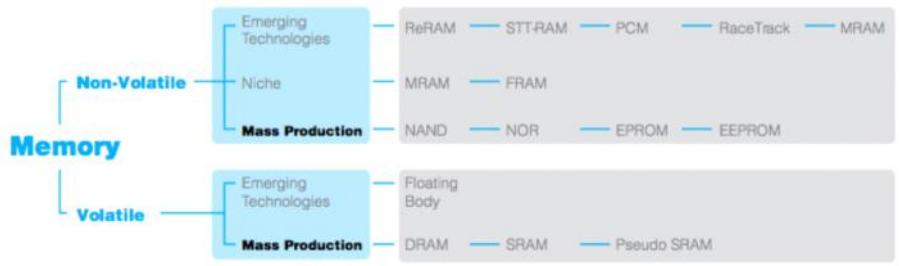
EEPROM & FLASH are in-system programmable



hid, Perkowski

Modern/New Memory Types

Saturday, March 5, 2022 12:46 PM



FeRAM: stores data in ferroelectric material.

No refresh, similar performance to DRAM but destructive reads

STT-RAM: Stores data in tunneled electrons. Value determined by electron spin.

High endurance, fast reads but high write energy

PCM: Stores data by changing state of material using a heater.

Good scalability but slow writes, low endurance.

ReRAM: Stores data by changing the resistance of a semiconductor.

Fast read/write, high density but limited endurance

Comparison of New vs Traditional Memories

Saturday, March 5, 2022 12:58 PM

Comparison of Traditional vs New memories:

STT-RAM: SRAM cache replacement

PCRAM: DRAM main memory and storage

ReRAM: NAND flash, embedded NOR flash

↳ also for near-memory compute

Features	SRAM	eDRAM	STT-RAM	PCRAM	ReRAM
Density	Low	High	High	Very high	Very high
Speed	Very Fast	Fast	Fast for read; slow for write	Slow for read; very slow for write	Slow for read/write
Dynamic Power	Low	Medium	Low for read; very high for write	Medium for read; high for write	Medium for read; high for write
Leakage Power	High	Medium	Low	Low	Low
Non-volatility	No	No	Yes	Yes	Yes